portion. This feature is unlike in Figs. 2 and 3, where there is a definite change in the behavior of D_{opt}/L as the flow regime changes.

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Temperature distribution within vortices in the wake of a cylinder

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INTRODUCTION

The transport equations for the vorticity ω and temperature T of a line vortex which is diffusing into the surrounding (ambient temperature) fluid are

$$\frac{\partial \omega}{\partial t} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \omega}{\partial r}\right) \quad (1)$$

and

$$\frac{\partial T}{\partial t} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right). \quad (2)$$

At $t = 0$, the circulation Γ_0 and thermal energy Q_0 are concentrated along the axis of rotation. Solutions to (1) and (2) are

$$\omega = \frac{\Gamma_0}{4\pi\nu t}\exp\left(-\frac{r^2}{4\nu t}\right) \quad (3)$$

and

$$T = \frac{Q_0}{4\pi\rho c_p \alpha t}\exp\left(-\frac{r^2}{4\alpha t}\right), \quad (4)$$

respectively. Equation (3) is given in a number of texts, e.g. refs. [1, 2]. Equation (4) was given in ref. [3] in the context of a line source of heat instantaneously released into an infinite solid. At the vortex centre, the vorticity and temperature are (at time t)

$$\omega_c = \frac{\Gamma_0}{4\pi\nu t} \quad (5)$$

and

$$T_c = \frac{Q_0}{4\pi\rho c_p \alpha t}. \quad (6)$$

The distributions for ω and T can be re-written in normalised form

$$\frac{\omega}{\omega_c} = \exp\left[-0.693 Pr^{-1}\left(\frac{r}{R}\right)^2\right] \quad (7)$$

and

$$\frac{T}{T_c} = \exp\left[-0.693\left(\frac{r}{R}\right)^2\right], \quad (8)$$

where the half-radius R is given by

$$\frac{R^2}{4\alpha t} = 0.693. \quad (9)$$

The vorticity distribution for vortices in the laminar wake ($Re_d = 140$) behind a cylinder was indirectly measured by Okude and Matsui [4] and was found to be in reasonable agreement with equation (7). To our knowledge, equation (8) has not been verified experimentally. This is surprising

NOMENCLATURE

c_p	specific heat at constant pressure [J kg^{-1} °C^{-1}]
d	cylinder diameter [mm]
l	cylinder length [mm]
n_s	vortex shedding frequency [Hz or s^{-1}]
N_0	electrical power used for heating the cylinder [W]
Pr	molecular Prandtl number, ν/α
Q_0	initial thermal energy [J m^{-1}]
Re_d	Reynolds number, $U_1 d/\nu$
r	radial distance from the vortex centre
R	half-radius or radial distance from vortex centre to the location where $T = T_c/2$ [m]
t	time [s]
T	temperature relative to ambient [°C]
T_c	temperature at the vortex centre [°C]
U_1	free-stream velocity [m s^{-1}]
U_c	vortex convection velocity [m s^{-1}]
x'	streamwise distance from vortex centre [m]
x	streamwise co-ordinate measured from cylinder axis [m]
y	lateral co-ordinate measured from wake centreline [m]
y_c	y location of vortex centre [m].

Greek symbols

α	thermal diffusivity [m^2 s^{-1}]
Γ_0	initial circulation [m^2 s^{-1}]
λ	wavelength of vortex street [m]
ν	kinematic viscosity [m^2 s^{-1}]
ρ	density [kg m^{-3}]
ω	vorticity [s^{-1}]
ω_c	vorticity at the vortex centre [s^{-1}].

since temperature can be measured more reliably than vorticity. The verification of equation (8) would be equivalent to verifying equation (7), the two solutions being identical when $Pr = 1$. The verification of equations (7) and (8) would be useful for the purpose of modelling the thermal laminar wake of a circular cylinder. Perhaps more practically, this note highlights the possible use of temperature as a relatively accurate marker of the vortices.

EXPERIMENTAL CONDITIONS

Measurements were made in an open-return low-turbulence wind tunnel with a 2.4 m long square working section (350 × 350 mm). The wake was generated by a cylinder (stainless steel rod) of diameter $d = 1.6$ mm spanning the width of the working section. The bottom wall of the working section was adjusted to obtain a zero streamwise pressure gradient. The free-stream velocity $U_1 \approx 0.92$ m s^{-1} and the Reynolds number Re_d ($\equiv U_1 d/\nu$) was about 98. The free-stream turbulence level was approximately 0.05%. Measurements were done at $x/d = 2.5$, 5, 11 and 17. The cylinder was heated electrically; the nominal heating rate ($N_0 \approx 1.5$ W) was sufficiently small for temperature to be treated as a passive scalar at all measurement stations.

For the temperature measurement, a single cold wire (0.63 μm diameter Wollaston Pr–10% Rh) was used and was operated by an in-house constant current (0.1 mA) circuit. In order to respond to the instantaneous temperature over a very small spatial volume (ideally at one point) within the moving vortex, the wire was etched to a short length of about 0.2 mm and aligned parallel to the cylinder axis. The signal from the circuit was offset, amplified and then digitised using a 12-bit A/D converter on a personal computer. To achieve good temporal resolution, a sampling frequency of 10 kHz was chosen (yielding 110 samples per vortex street wavelength). A filter cut-off frequency of 500 Hz was used to eliminate amplifier noise. The vortex shedding frequency n_s (≈ 91 Hz) was monitored during the experiment using a real-time spectrum analyser (HP3582A).

The cold wire was mounted on a Mitutoyo height gauge (with a least count of 0.01 mm) for traversing across the wake. The wire was calibrated in the exit plane of a circular jet using a 10 Ω platinum resistance thermometer operated in a Leeds and Northrup 8087 bridge (with a resolution of 0.01°C).

Table 1. Location of vortex centre

x/d	2.5	5	11	17
$\lvert y_c \rvert/d$	0.41	0.5	0.62	0.65

RESULTS AND DISCUSSION

The vortex street wavelength λ ($\equiv U_c n_s^{-1}$), estimated by using a convection velocity U_c of about $0.85U_1$ [5], is about $5.4d$. To determine the distance y_c of the vortex centre from the wake centreline, the peak temperature was identified at each y position of the wire. The value of y at which the largest peak occurred was assumed to correspond to y_c. The values of y_c/d (Table 1; the uncertainty ranges from ±4% at $x/d = 2.5$ to ±6% at $x/d = 17$) are generally consistent with those of Schaefer and Eskinazi [6] for $Re_d = 94$.

Temperature signals at $y = y_c$ are shown in Fig. 1. The primary peak at $x' = 0$ is associated with the temperature T_c at the vortex centres, while the secondary peaks are induced by the neighbouring vortices at $y = -y_c$. The relatively strong engulfment of cold free-stream fluid on the downstream side of each vortex lowers the temperature by comparison to that on the upstream side, thus accounting for the slight asymmetry about $x' = 0$ at relatively large values of $|x'|$.

The temperature distributions in both x and y directions are plotted in Fig. 2, in the form $\log_{10}(T/T_c)$ vs $(r/R_x)^2$ where R_x, R_y denote the half-radii in the x and y directions,

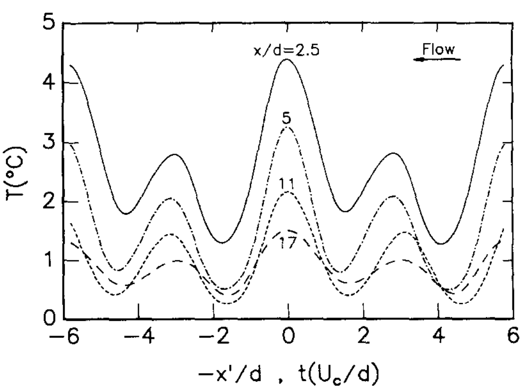

FIG. 1. Temperature signals at the vortex centres. ——, $x/d = 2.5$; —·—, 5; ----, 11; – – –, 17.

respectively. (The uncertainty in T_c ranges from 0.5% at $x/d = 2.5$ to 1.4% at $x/d = 17$.) For comparison, equation (8) is also shown (solid straight line) in this figure. Except at $x/d = 2.5$, the agreement between measurement and theory is excellent for $r \leqslant 1.2R$. (The departure at $x/d = 2.5$ appears to be caused by the early stages of vortex formation immediately downstream of the cylinder.) By contrast, the data for ω/ω_c obtained by Okude and Matsui [4] for $Re_d = 140$, provide less satisfactory approximations to equation (7). There are likely error sources in estimating ω: the streamwise velocity derivative relies on the transformation $x = -U_c t$ while the measurement of the lateral velocity derivative is rather delicate.

The streamwise dependence of T_c is shown in Fig. 3. The data for T_c^{-1} follow a straight line, which intersects the x axis at $x/d = -5.5$; this latter position may be identified with the virtual origin of the wake vortices. The initial thermal energy Q_0 of the vortices

$$Q_0 = \frac{N_0}{2n_s l}, \qquad (10)$$

where l (= 0.4 m) is the cylinder length. With $N_0 \simeq 1.5$ W, $Q_0 \simeq 0.0206$ J m^{-1} and, after substitution in equation (6), $T_c \simeq 0.062t^{-1}$ (°C), which is indicated by a solid line in Fig. 3. The experimental data are in agreement with this line, implying that the spatial evolution of temperature at the vortex centre complies reasonably well with the temporal evolution of equation (6). The small difference between theory and measurements is possibly due to Q_0 being slightly overestimated by equation (10), mainly because of cylinder end losses and conduction to the walls of the wind tunnel.

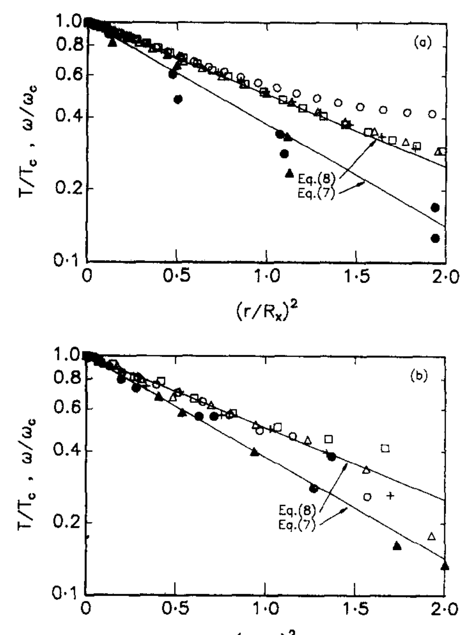

FIG. 2. Comparison between the temperature T (or vorticity ω) within the wake vortices and equation (8) (or equation (7)). (a) x direction; (b) y direction. T (present): ○, $x/d = 2.5$; △, 5; +, 11; □, 17. ω (Okude and Matsui [4]): ▲, $x/d = 15$; ●, 20.

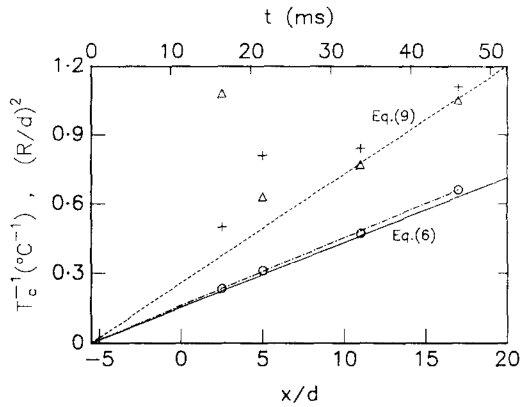

FIG. 3. Streamwise variations of temperature at the vortex centre and the vortex half-radius. ○, T_c^{-1}; △, R_x^2; +, R_y^2. —·—, best fit to T_c^{-1}; ——, equation (6); ---, equation (9).

Figure 3 indicates that R_x^2 is significantly greater than R_y^2 at $x/d = 2.5$, while the inverse applies at $x/d = 5$; at $x/d = 11$ and 17, R_y^2 remains (slightly) greater than R_x^2. This trend suggests that the vortices are elliptical (in shape) in the region immediately behind the cylinder. Further downstream, the difference between the magnitudes of the major and minor axes gradually disappears, the vortices becoming nearly axisymmetrical at $x/d \gtrsim 11$. Beyond this station, the measured half-radius appears to be in reasonable agreement with equation (9).

CONCLUSIONS

The present results indicate that the temperature distribution within laminar vortices in the wake of a circular cylinder is quite well approximated by the theoretical distribution for a diffusing line vortex. The approximation is less adequate immediately downstream of the cylinder, where the vortices are not axisymmetrical in shape. The streamwise variation of temperature at the vortex centre can be inferred, with relatively good accuracy, from the temporal temperature variation for an isolated line vortex.

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